General treatment of isocurvature perturbations and non-Gaussianities

David Langlois[a] and Angela Lepidi[a,b,c]

[a] APC (CNRS-Université Paris 7),
10, rue Alice Domon et Léonie Duquet, 75205 Paris Cedex 13, France

[b] Istituto Nazionale di Fisica Nucleare, Sezione di Ferrara, I-44100 Ferrara, Italy

[c] Dipartimento di Fisica, Università degli Studi di Ferrara, I-44100 Ferrara, Italy

December 14, 2010

Abstract

We present a general formalism that provides a systematic computation of the linear and non-linear perturbations for an arbitrary number of cosmological fluids in the early Universe going through various transitions, in particular the decay of some species (such as a curvaton or a modulus). Using this formalism, we revisit the question of isocurvature non-Gaussianities in the mixed inflaton-curvaton scenario and show that one can obtain significant non-Gaussianities dominated by the isocurvature mode while satisfying the present constraints on the isocurvature contribution in the observed power spectrum. We also study two-curvaton scenarios, taking into account the production of dark matter, and investigate in which cases significant non-Gaussianities can be produced.

1 Introduction

In many occasions, cosmology has been and still is an invaluable means to constrain particle physics models. These constraints can arise by using information from homogeneous cosmology, such as the constraints on the light degrees of freedom at nucleosynthesis. With the discovery of the Cosmic Microwave Background (CMB) fluctuations, new constraints arise from the observed power spectrum of linear perturbations. Even more recently, the

upper bounds on primordial non-Gaussianities have started to be used to constrain early Universe scenarios.

Although the simplest early Universe models are based on inflationary models with a single scalar field, many models involve additional scalar fields, which can play a dynamical role during inflation or simply be spectactor fields (see e.g. [1] for introductory lectures). The existence of several degrees of freedom opens up the possibility of isocurvature perturbations, i.e. perturbations in the particle density ratio between two fluids, for example cold dark matter (CDM) isocurvature perturbations (between CDM and radiation) or baryon isocurvature perturbations (between baryons and radiation). Since primordial isocurvature perturbations leave distinctive features in the CMB anisotropies, they can be in principle disentangled from the usual adiabatic mode. The present upper bound on the isocurvature contribution to the power spectrum provides a stringent constraint.

This is the case for the curvaton scenario [2] where large residual isocurvature perturbations (for CDM or baryons) can be generated, depending on how and when CDM or baryons are produced [3, 4] (see also [5, 6] for more detailed scenarios). The same constraints apply to moduli that are light during inflation, and thus acquire super-Hubble fluctuations, as discussed recently in [7].

Another potentially useful information on primordial perturbations is the amplitude and shape of their non-Gaussianity. So far, the current CMB data seem to favour a non-zero amount of so-called local non-Gaussianity [8], but Planck data will be needed to confirm or infirm this trend. Several models can generate local non-Gaussianity (see e.g. [9] for a recent review): multiple field inflation (during inflation or at the end of inflation: see e.g. [10]), modulated reheating [11, 12], curvaton, modulated trapping [13], etc. It is thus interesting to combine the constraints on isocurvature modes and non-Gaussianity to explore the early Universe physics, as has been done recently in various scenarios [14, 15, 16, 17, 18, 19, 20, 21].

The purpose of the present work is to give a unified treatment of linear and nonlinear perturbations, which enables to compute their evolution through one or several cosmological transitions, such as the decay of some particle species. Our treatment takes into account the various decay products and their branching ratio. Our formalism can thus be applied to a large class of early Universe scenarios, in order to compute automatically their predictions for adiabatic and isocurvature perturbations, and their non-Gaussianities. As input, one simply needs parameters that depend on the homogeneous evolution. This thus provides a simple way to confront an early Universe scenario, and its underlying particle physics model, with the present and future cosmological data.

As applications to our general formalism, we consider two specific examples. The first example is a more refined treatment of the isocurvature perturbations and their non-Gaussianity in the mixed curvaton-inflation scenario [22, 23, 24]. The second example deals with a multiple-curvaton scenario [25, 26, 27, 28]. In both examples, we generalize the results that have been obtained in previous works, allowing the curvaton to decay into several species.

This paper is organized as follows. In Section 2, we introduce the non-linear curvature and isocurvature perturbations. Section 3 is devoted to the general treatment of a cosmological transition, such as the decay of some particle species. In Section 4, we focus on the first application, namely the mixed curvaton-inflaton scenario with a single curvaton. In Section 5, we consider scenarios with two curvatons. We conclude in the final Section.

2 Non-linear curvature perturbations

We first introduce the notion of non-linear curvature perturbation. Several definitions have been proposed, which turn out to be equivalent on large scales, and we will follow here the covariant approach introduced in [29, 30], and reviewed recently in [31].

For a perfect fluid characterized by the energy density ρ, the pressure P and the four-velocity u^a, the conservation law $\nabla_a T^a_b = 0$ for the energy-momentum tensor, $T_{ab} = (\rho + P) u_a u_b + P g_{ab}$, implies that the covector

$$\zeta_a \equiv \nabla_a N - \frac{\dot{N}}{\dot{\rho}} \nabla_a \rho \tag{1}$$

satisfies the relation

$$\dot{\zeta}_a \equiv \mathcal{L}_u \zeta_a = -\frac{\Theta}{3(\rho + p)} \left(\nabla_a p - \frac{\dot{p}}{\dot{\rho}} \nabla_a \rho \right) , \tag{2}$$

with the definitions

$$\Theta \equiv \nabla_a u^a, \quad N \equiv \frac{1}{3} \int d\tau \, \Theta , \tag{3}$$

where τ is the proper time along the fluid worldlines and a dot denotes a Lie derivative along u^a, which is equivalent to an ordinary derivative for *scalar* quantities (e.g. $\dot{\rho} \equiv u^a \nabla_a \rho$). N can be interpreted as the number of e-folds of the local scale factor associated with an observer following the fluid.

The covector ζ_a can be defined for the global cosmological fluid or for any of the individual cosmological fluids, as long as they are non-interacting

(the case of interacting fluids is discussed in [32]). Using the non-linear conservation equation

$$\dot{\rho} = -3\dot{N}(\rho + P) , \tag{4}$$

which follows from $u^b \nabla_a T^a_b = 0$, one can re-express ζ_a in the form

$$\zeta_a = \nabla_a N + \frac{\nabla_a \rho}{3(\rho + P)} . \tag{5}$$

If $w \equiv P/\rho$ is constant, the above covector is a total gradient and can be written as

$$\zeta_a = \nabla_a \left[N + \frac{1}{3(1+w)} \ln \rho \right] . \tag{6}$$

On scales larger than the Hubble radius, the above definitions are equivalent to the non-linear curvature perturbation on uniform density hypersurfaces as defined in [33],

$$\zeta = \delta N - \int_{\bar{\rho}}^{\rho} H \frac{d\tilde{\rho}}{\dot{\tilde{\rho}}} = \delta N + \frac{1}{3} \int_{\bar{\rho}}^{\rho} \frac{d\tilde{\rho}}{(1+w)\tilde{\rho}} , \tag{7}$$

where $H = \dot{a}/a$ is the Hubble parameter.

It will be useful to distinguish the non-linear curvature perturbation ζ of the total fluid, from the individual non-linear perturbation ζ_A that describes the cosmological fluid A (with $w_A \equiv P_A/\rho_A = 0$ for a pressureless fluid or $w_A = 1/3$ for a relativistic fluid), defined by

$$\zeta_A = \delta N + \frac{1}{3(1+w_A)} \ln \left(\frac{\rho_A}{\bar{\rho}_A} \right) , \tag{8}$$

where a bar denotes a homogeneous quantity.

Inverting this relation yields the expression of the inhomogeneous energy density as a function of the background energy density and of the curvature perturbation ζ_A,

$$\rho_A = \bar{\rho}_A e^{3(1+w_A)(\zeta_A - \delta N)} , \tag{9}$$

which we will use many times in the following.

The non-linear isocurvature (or entropy) perturbation between two fluids A and B is defined by

$$S_{A,B} \equiv 3(\zeta_A - \zeta_B). \tag{10}$$

In the following, we will always define the isocurvature perturbations with respect to the radiation fluid, so that our definition for the isocurvature perturbation of the fluid A will be

$$S_A \equiv 3(\zeta_A - \zeta_r), \tag{11}$$

where ζ_r is the uniform-density curvature perturbation of the radiation fluid.

3   Decay

Let us now consider a cosmological transition associated with the decay of some species of particles, which we will call σ.

In the sudden decay approximation, the decay takes place on the hypersurface characterized by the condition

$$H_{\mathrm{d}} = \Gamma_\sigma . \tag{12}$$

Therefore, since H depends only on the *total* energy density, the decay hypersurface is a hypersurface of uniform total energy density, with $\delta N_{\mathrm{d}} = \zeta$, where ζ is the global curvature perturbation. Using (9), the equality between the total energy densities, respectively before and after the decay, thus reads

$$\sum_A \bar\rho_{A-} e^{3(1+w_A)(\zeta_{A-}-\zeta)} = \bar\rho_{\mathrm{decay}} = \sum_B \bar\rho_{B+} e^{3(1+w_B)(\zeta_{B+}-\zeta)}, \tag{13}$$

where the subscripts $-$ and $+$ label quantities defined, respectively, *before* and *after* the transition.

3.1   Before the decay

The first equality in (13) leads to

$$\sum_A \Omega_A e^{3(1+w_A)(\zeta_{A-}-\zeta)} = 1, \tag{14}$$

where we have defined $\Omega_A \equiv \bar\rho_{A-}/\bar\rho_{\mathrm{decay}}$ (to avoid confusion, the Ω_A are always defined just *before* the decay). The above relation determines ζ as a function of the ζ_{A-}.

At linear order, this gives

$$\zeta = \frac{1}{\tilde\Omega} \sum_A \tilde\Omega_A \zeta_{A-} \qquad \text{(first order)} \tag{15}$$

with the notation

$$\tilde\Omega_A \equiv (1+w_A)\Omega_A, \qquad \tilde\Omega \equiv \sum_A \tilde\Omega_A . \tag{16}$$

Expanding (14) up to second order, one finds

$$\zeta = \frac{1}{\tilde\Omega} \sum_A \tilde\Omega_A \left[\zeta_{A-} + \frac{3}{2}(1+w_A)\left(\zeta_{A-} - \zeta\right)^2 \right] \qquad \text{(second order)} \tag{17}$$

where, on the right hand side, ζ is to be replaced by its first order expression (15).

5

3.2 After the decay

We now consider the outcome of the decay. In general, the species σ decays into various species A, with respective decay widths $\Gamma_{A\sigma}$. Defining the relative branching ratios

$$\gamma_{A\sigma} \equiv \frac{\Gamma_{A\sigma}}{\Gamma_\sigma}, \quad \Gamma_\sigma \equiv \sum_A \Gamma_{A\sigma}, \tag{18}$$

one can write the energy density of the fluid A after the decay in terms of the energy densities of A and of σ as

$$\rho_{A+} = \rho_{A-} + \gamma_{A\sigma}\rho_{\sigma-}. \tag{19}$$

Using (9), one can rewrite this nonlinear equation in terms of the curvature perturbations ζ_{A+}, ζ_{A-} and $\zeta_{\sigma-}$, which yields

$$e^{3(1+w_A)(\zeta_{A+}-\zeta)} = \frac{\bar{\rho}_{A-}e^{3(1+w_A)(\zeta_{A-}-\zeta)} + \gamma_{A\sigma}\bar{\rho}_{\sigma-}e^{3(1+w_\sigma)(\zeta_\sigma-\zeta)}}{\bar{\rho}_{A-} + \gamma_{A\sigma}\bar{\rho}_{\sigma-}}. \tag{20}$$

This expression thus gives ζ_{A+} as a function of ζ_{A-}, ζ_σ and of the global ζ. Substituting ζ in terms of ζ_σ and of all the ζ_{B-}, one finally obtains ζ_{A+} as a function of all the ζ_{B-}.

Following this procedure, one finds that the linear curvature perturbation for any given fluid A is given by

$$\zeta_{A+} = \sum_B T_A{}^B \zeta_{B-} \qquad \text{(first order)} \tag{21}$$

with

$$T_A{}^A = 1 - f_A + f_A \frac{(w_A - w_\sigma)\tilde{\Omega}_A}{(1+w_A)\tilde{\Omega}} \tag{22}$$

$$T_A{}^\sigma = f_A \frac{1+w_\sigma}{1+w_A} + f_A \frac{(w_A - w_\sigma)\tilde{\Omega}_\sigma}{(1+w_A)\tilde{\Omega}} \tag{23}$$

$$T_A{}^C = f_A \frac{(w_A - w_\sigma)\tilde{\Omega}_C}{(1+w_A)\tilde{\Omega}}, \qquad C \neq A, \sigma. \tag{24}$$

In the above expressions, we have introduced the parameter

$$f_A \equiv \frac{\gamma_{A\sigma}\Omega_\sigma}{\Omega_A + \gamma_{A\sigma}\Omega_\sigma}, \tag{25}$$

which represents the fraction of the fluid A that has been created by the decay. If A does not belong to the decay products of σ, then $f_A = 0$. The

opposite limit, $f_A = 1$, occurs when all the fluid A is produced by the decay. For the intermediate values of f_A, part of A is produced by the decay while the other part is preexistent.

In the following, we will assume that the decaying species behaves like non-relativistic matter (this is the case for a curvaton or modulus field that oscillates in a quadratic potential) and we will thus always use $w_\sigma = 0$.

From the above expressions (22-24), it is straightforward to check that

$$\sum_B T_A{}^B = 1. \tag{26}$$

The post-decay perturbation ζ_{A+} can thus be seen as the barycenter of the pre-decay perturbations ζ_{B-} with the weights $T_A{}^B$ (all these coefficients satisfy $0 \le T_A{}^B \le 1$ for $w_\sigma = 0$). Note that if the fluid A is not produced in the decay (i.e. $f_A = 0$), then the transfer coefficients are trivial: $T_A{}^B = \delta_A{}^B$.

Since it is convenient to use the same range of species indices *before and after* the transition, we also introduce the coefficients $T_\sigma{}^B = 0$, which imply that $\zeta_{\sigma+} = 0$. This convention will be especially useful when one needs to combine several transitions, as we will discuss soon.

At second order, expanding (20) and substituting the first order expression (15) for ζ, one obtains

$$\zeta_{A+} = \sum_B T_A{}^B \zeta_{B-} + \sum_{B,C} U_A^{BC} \zeta_{B-} \zeta_{C-}, \qquad \text{(second order)} \tag{27}$$

with

$$U_A^{BC} \equiv \frac{3}{2} \left[T_{AB}(1+w_B)\delta_{BC} + 2\frac{\tilde{\Omega}_C}{\tilde{\Omega}}(w_A - w_B)T_{AB} - (1+w_A)T_{AB}T_{AC} \right.$$
$$\left. - \frac{\tilde{\Omega}_B \tilde{\Omega}_C}{\tilde{\Omega}^2} \left(1 + w_A - \sum_D T_{AD}(1+w_D) \right) \right]. \tag{28}$$

The change of the various isocurvature perturbations, defined in (11), can also be determined by using the above expressions. In particular, at linear order, one finds, using the property (26), the simple expression

$$S_{A+} = \sum_B \left(T_A{}^B - T_r{}^B \right) S_{B-} \qquad \text{(first order)}. \tag{29}$$

3.3 Several transitions

If the early Universe scenario involves several cosmological transitions, for example several particle decays, one can use the above expressions successively

to determine the final "primordial" perturbations, i.e. the initial conditions at the onset of the standard cosmological era.

For linear perturbations, the expression of the final perturbations as a function of the initial ones, is simply given by

$$\zeta_A^{(f)} = \sum_B T_A{}^B \zeta_B^{(i)}, \qquad T = \prod_k T_{[k]} \tag{30}$$

where T is the matricial product of all transfer matrices $T_{[k]}$, which describe the successive transitions.

The cosmological transitions can result from the decay of some particle species but they can be of other types. For example, if CDM consists of WIMPs (Weakly Interacting Massive Particles), the freeze-out can be treated as a cosmological transition. If radiation is the dominant species at freeze-out, then $\zeta_{c+} = \zeta_r$. But, if other species are significant in the energy budget of the universe at the time of freeze-out, any entropy perturbation between the extra species and radiation will modify the above relation. The presence of a pressureless component, like a curvaton, leads to [4]

$$\zeta_{c+} = \zeta_{r-} + \frac{(\alpha_f - 3)\Omega_\sigma}{2(\alpha_f - 2) + \Omega_\sigma}\left(\zeta_{\sigma-} - \zeta_{r-}\right), \qquad \alpha_f \equiv \frac{m_c}{T_f} + \frac{3}{2} \tag{31}$$

at linear order, while the other ζ_A remain unchanged. The symbol "σ" denotes here the conglomerate of all pressureless matter at the time of freeze-out, except of course the CDM species that is freezing out.

4   Scenario with a single curvaton

Let us now apply our formalism to a simple scenario with only three initial species: radiation (r), CDM (c) and a curvaton (σ), considered in e.g. [34]. After the decay of the curvaton, the radiation and CDM perturbations remain unchanged and provide the initial conditions for the perturbations at the onset of the standard cosmological phase (let us say around $T \sim 1$ MeV).

4.1   Perturbations after the decay

4.1.1   Linear order

According to the expressions (22-24), the linear transfer matrix T_{AB} is given in this case by

$$T = \begin{pmatrix} 1 - x_r & x_c & x_r - x_c \\ 0 & 1 - f_c & f_c \\ 0 & 0 & 0 \end{pmatrix}, \qquad x_r \equiv \frac{f_r}{\widetilde{\Omega}}, \qquad x_c \equiv \frac{1}{4}\Omega_c\, x_r \tag{32}$$

where the order of the species is (r, c, σ). This means that the linear curvature perturbations for radiation and for CDM, after the curvaton decay, are given respectively by

$$\zeta_{r+} = (1 - x_r)\,\zeta_{r-} + x_c\,\zeta_{c-} + (x_r - x_c)\,\zeta_{\sigma-} \tag{33}$$

and

$$\zeta_{c+} = (1 - f_c)\,\zeta_{c-} + f_c\,\zeta_{\sigma-}. \tag{34}$$

The entropy perturbation after the decay is thus

$$\frac{1}{3}S_{c+} \equiv \zeta_{c+} - \zeta_{r+} = (1 - f_c - x_c)\zeta_{c-} + (x_r - 1)\zeta_{r-} + (f_c + x_c - x_r)\zeta_{\sigma-}\,, \tag{35}$$

which can also be expressed directly in terms of the pre-decay entropy perturbations, following (29),

$$S_{c+} = (1 - f_c - x_c)\,S_{c-} + (f_c + x_c - x_r)\,S_{\sigma-}\,. \tag{36}$$

Note that, if many CDM particles are created by the decay of the curvaton, a significant fraction of them could annihilate, leading to an effective suppression of the final isocurvature perturbation. This effect has been studied in [5] and can easily be incorporated in our formalism.

In practice, we will need the above expressions only in the limit $x_c = 0$ since Ω_c is usually negligible when the decay occurs. The coefficient x_r, which we will shorten into r from now on, can then be expressed as

$$r \equiv x_r = \frac{f_r}{\Omega_\sigma}\left(\frac{3\Omega_\sigma}{4 - \Omega_\sigma}\right) \equiv \xi\,\tilde{r}. \tag{37}$$

The first factor,

$$\xi \equiv \frac{f_r}{\Omega_\sigma} = \frac{\gamma_{r\sigma}}{1 - (1 - \gamma_{r\sigma})\Omega_\sigma} \tag{38}$$

can be interpreted as the transfer efficiency between the curvaton and radiation. Its maximal value, $\xi = 1$, is reached when all the energy stored in the curvaton is converted into radiation, i.e. when $\gamma_{r\sigma} = 1$, as usually assumed in most works on the curvaton. However, if a fraction of the curvaton energy goes into species other than radiation, then the transfer efficiency ξ is reduced. The second factor,

$$\tilde{r} \equiv \frac{3\,\Omega_\sigma}{4 - \Omega_\sigma}, \tag{39}$$

is the familiar coefficient that appears in the literature on the curvaton, which coincides with our r only if $\xi = 1$.

9

4.1.2 Second order

The expressions for the curvature perturbations up to second order are obtained from the general expression (27-28), using the transfer matrix (32). The expression for CDM is relatively simple:

$$\zeta_{c+} = (1 - f_c)\zeta_{c-} + f_c\zeta_{\sigma-} + \frac{3}{2}f_c(1 - f_c)\left(\zeta_{c-} - \zeta_{\sigma-}\right)^2 . \tag{40}$$

The expression for radiation is much more complicated in general, but in the limit $x_c = 0$, which is of interest to us, the radiation perturbation reduces to

$$\zeta_{r+} = \zeta_{r-} + \frac{r}{3}S_{\sigma-} + \frac{r}{18}\left[3 - 4r + \frac{2r}{\xi} - \frac{r^2}{\xi^2}\right]S_{\sigma-}^2 . \tag{41}$$

In the limit $\gamma_{r\sigma} = 1$, i.e. $\xi = 1$, one recovers the usual expression.

Note that, although Ω_c is assumed to be very small, it cannot be neglected in the expression for f_c [see (25)] because $\gamma_{c\sigma}$ or Ω_σ can be very small, and f_c can thus take any value between 0 and 1.

4.2 Initial curvaton perturbation

We now need to relate the perturbation of the curvaton fluid with the fluctuations of the curvaton scalar field during inflation. For simplicity, we assume here that the potential of the curvaton is quadratic.

Before its decay, the oscillating curvaton (with mass $m \gg H$) is described by a pressureless, non-interacting fluid with energy density

$$\rho_\sigma = m^2\sigma^2 , \tag{42}$$

where σ is the rms amplitude of the curvaton field. Making use of Eq. (9), the inhomogeneous energy density of the curvaton can be reexpressed as

$$\rho_\sigma = \bar{\rho}_\sigma e^{3(\zeta_\sigma - \delta N)} . \tag{43}$$

In the post-inflation era where the curvaton is still subdominant, the spatially flat hypersurfaces are characterized by $\delta N = \zeta_r$ (since CDM is also subdominant). On such a hypersurface, the curvaton energy density can be written as

$$\bar{\rho}_\sigma e^{3(\zeta_\sigma - \zeta_r)} = \bar{\rho}_\sigma e^{S_\sigma} = m^2\left(\bar{\sigma} + \delta\sigma\right)^2 . \tag{44}$$

Expanding this expression up to second order, and using the conservation of $\delta\sigma/\sigma$ in a quadratic potential, we obtain

$$S_\sigma = 2\frac{\delta\sigma_*}{\bar{\sigma}_*} - \left(\frac{\delta\sigma_*}{\bar{\sigma}_*}\right)^2 , \tag{45}$$

10

where the initial curvaton field perturbation, $\delta\sigma_*$, is assumed to be Gaussian, as would be expected for a weakly coupled field. The curvaton entropy perturbation (45) thus contains a linear part $\hat{S}$ which is Gaussian and a second order part which is quadratic in $\hat{S}$:

$$S_\sigma = \hat{S} - \frac{1}{4}\hat{S}^2, \quad \text{with} \quad \hat{S} \equiv 2\frac{\delta\sigma_*}{\bar{\sigma}_*} \tag{46}$$

where the power spectrum of $\hat{S}$, generated during inflation, is given by

$$\langle \hat{S}(\vec{k})\hat{S}(\vec{k}')\rangle = (2\pi)^3 \frac{2\pi^2}{k^3} \mathcal{P}_{\hat{S}}(k)\, \delta(\vec{k} + \vec{k}'), \quad \mathcal{P}_{\hat{S}}(k) = \frac{4}{\sigma_*^2}\left(\frac{H_*}{2\pi}\right)^2 . \tag{47}$$

The subscript $*$ means that the quantity is evaluated at the time when the corresponding scale crossed out the Hubble radius during inflation.

4.3 Primordial adiabatic and isocurvature perturbations

For simplicity, we now assume that the post-inflation perturbations for dark matter and radiation, i.e. before the curvaton decay, are the same and depend only on the inflaton fluctuations,

$$\zeta_{c-} = \zeta_{r-} = \zeta_{\text{inf}}, \tag{48}$$

so that there are only two independent degrees of freedom from the inflationary epoch, ζ_{inf} and $\hat{S}$.

Substituting (46) and (48) into (41) and (40) yields

$$\zeta_r = \zeta_{\text{inf}} + \frac{r}{3}\hat{S} + \frac{r}{36}\left[3 - 8r + \frac{4r}{\xi} - 2\frac{r^2}{\xi^2}\right]\hat{S}^2 \tag{49}$$

and

$$S_c = (f_c - r)\hat{S} + \frac{1}{12}\left[3f_c(1 - 2f_c) - r\left(3 - 8r + \frac{4r}{\xi} - 2\frac{r^2}{\xi^2}\right)\right]\hat{S}^2, \tag{50}$$

In the limit $\gamma_{r\sigma} = 1$, i.e. $\xi = 1$, one recovers the well-known expression for ζ_r.

Considering only the linear part of (49), one finds that the power spectrum for the primordial adiabatic perturbation ζ_r can be expressed as

$$\mathcal{P}_{\zeta_r} = \mathcal{P}_{\zeta_{\text{inf}}} + \frac{r^2}{9}\mathcal{P}_{\hat{S}} \equiv (1 + \lambda)\mathcal{P}_{\zeta_{\text{inf}}} \equiv \Xi^{-1}\frac{r^2}{9}\mathcal{P}_{\hat{S}} \tag{51}$$

11

where λ is defined as the ratio between the curvaton and inflaton contributions and $\Xi = (1+\lambda^{-1})^{-1}$ as the ratio between the curvaton contribution and the total curvature power spectrum. The limit $\lambda \gg 1$, or $\Xi \simeq 1$, corresponds to the standard curvaton scenario, where the inflaton perturbation is ignored. The cases where the inflaton contribution is not negligible correspond to the mixed inflaton-curvaton scenario [22]. The curvaton contribution is subdominant when $\lambda \ll 1$, i.e. $\Xi \ll 1$.

Let us now turn to the primordial isocurvature perturbation. As can be read from the linear part of (50), its power spectrum is given by

$$\mathcal{P}_{S_c} = (f_c - r)^2 \mathcal{P}_{\hat{S}} \,. \tag{52}$$

and the correlation between adiabatic and isocurvature fluctuations is

$$\mathcal{C} \equiv \frac{\mathcal{P}_{S_c,\zeta_r}}{\sqrt{\mathcal{P}_{S_c}\mathcal{P}_{\zeta_r}}} = \varepsilon_f \, \Xi^{1/2} \,, \qquad \varepsilon_f \equiv \mathrm{sgn}(f_c - r) \,. \tag{53}$$

In the pure curvaton limit ($\Xi \simeq 1$), adiabatic and isocurvature perturbations are either fully correlated, if $\varepsilon_f > 0$, or fully anti-correlated, if $\varepsilon_f < 0$. In the opposite limit ($\Xi \ll 1$), the correlation vanishes. For intermediate values of Ξ, the correlation is only partial, as can be also obtained in multifield inflation [35].

As combined adiabatic and isocurvature perturbations lead to a distortion of the acoustic peaks, which depends on their correlation [36], it is in principle possible to distinguish, in the observed fluctuations, the adiabatic and isocurvature contributions. So far, there is no detection of any isocurvature component, but only an upper bound on the ratio between isocurvature and adiabatic power spectra, which, in our case, is given by

$$\alpha \equiv \frac{\mathcal{P}_{S_c}}{\mathcal{P}_{\zeta_r}} = 9 \left(1 - \frac{f_c}{r} \right)^2 \Xi \,. \tag{54}$$

The observational constraints on α depend on the correlation. Writing $\alpha \equiv a/(1-a)$ (note that $\alpha \simeq a$ if α is small), the constraints (WMAP+BAO+SN) given in [8] are

$$a_0 < 0.064 \quad (95\%\mathrm{CL}), \qquad a_1 < 0.0037 \quad (95\%\mathrm{CL}) \tag{55}$$

respectively for the uncorrelated case and for the fully correlated case [1].

One sees that the observational constraint $\alpha \ll 1$ can be satisfied in only two cases:

---

[1]Our notations differ from those of [8]. Our a corresponds to their α and our fully *correlated* limit corresponds to their fully *anti-correlated* limit, because their definition of the correlation has the opposite sign.

- $|f_c - r| \ll r$, i.e. a fine-tuning between the two parameters f_c and r. This includes the case $f_c = 1$ with $r \simeq 1$, considered in [17].

- $\Xi \ll 1$, i.e. the curvaton contribution to the observed power spectrum is very small.

4.4 Non-Gaussianities

Let us now examine the amplitude of the non-Gaussianities that can be generated in our model. We recall that our observable quantities ζ and S are of the form

$$\zeta = \zeta_{\text{inf}} + z_1 \hat{S} + \frac{1}{2} z_2 \hat{S}^2, \qquad S = s_1 \hat{S} + \frac{1}{2} s_2 \hat{S}^2, \tag{56}$$

where ζ_{inf} and $\hat{S}$ are two independent Gaussian fields and where the coefficients can be read explicitly from (49) and (50).

Applying the general results of the Appendix to the present situation, we can easily compute for our model the "reduced" angular bispectrum, which is of direct interest for a comparison with CMB observations and which generalizes the analysis of [37] in the purely adiabatic case. Specializing (112) to our case, one finds

$$b_{l_1 l_2 l_3} = 3 \sum_{I,J,K} b_{NL}^{I,JK} \int_0^\infty r^2 dr\, \tilde{\beta}_{(l_1}^I(r) \beta_{l_2}^J(r) \beta_{l_3)}^K(r) \tag{57}$$

with

$$b_{NL}^{I,JK} \equiv N_{(2)}^I N_{(1)}^J N_{(1)}^K, \tag{58}$$

where $N_{(2)}^\zeta = z_2$, $N_{(2)}^S = s_2$, $N_{(1)}^\zeta = z_1$, $N_{(1)}^S = s_1$, respectively, and

$$\tilde{\beta}_l^I(r) \equiv \frac{2}{\pi} \int k^2 dk\, j_l(kr) g_l^I(k), \quad \beta_l^I(r) \equiv \frac{2}{\pi} \int k^2 dk\, j_l(kr) g_l^I(k) P_{\hat{S}}(k), \tag{59}$$

where the $g_l^I(k)$ denote the adiabatic ($I = \zeta$) and isocurvature ($I = S$) transfer functions. Because of the symmetry under exchange of the last two indices, the reduced bispectrum contains six different contributions, whose amplitude is parametrized by the six coefficients $b_{NL}^{I,JK}$.

In order to compare these coefficients with the usual parameter f_{NL} defined in the purely adiabatic case, one must recall that f_{NL} is proportional to the bispectrum of ζ divided by the square of its power spectrum. By noting that the $\beta_l^I(r)$ introduced in (59) involve $P_{\hat{S}}$, this implies that the analogs

13

of f_{NL} can be defined by dividing the coefficient $b_{NL}^{I,JK}$ by the square of the ratio $P_\zeta/P_{\hat{S}} = z_1^2 \Xi^{-1}$. We thus introduce the parameters

$$\tilde{f}_{NL}^{I,JK} \equiv \frac{6}{5} f_{NL}^{I,JK} \equiv \frac{\Xi^2}{z_1^4} b_{NL}^{I,JK}, \tag{60}$$

explicitly given by the expressions

$$\tilde{f}_{NL}^{\zeta,\zeta\zeta} = \frac{z_2}{z_1^2}\Xi^2, \quad \tilde{f}_{NL}^{\zeta,\zeta S} = \frac{s_1 z_2}{z_1^3}\Xi^2, \quad \tilde{f}_{NL}^{\zeta,SS} = \frac{s_1^2 z_2}{z_1^4}\Xi^2, \tag{61}$$

$$\tilde{f}_{NL}^{S,\zeta\zeta} = \frac{s_2}{z_1^2}\Xi^2, \quad \tilde{f}_{NL}^{S,\zeta S} = \frac{s_1 s_2}{z_1^3}\Xi^2, \quad \tilde{f}_{NL}^{S,SS} = \frac{s_1^2 s_2}{z_1^4}\Xi^2. \tag{62}$$

In the absence of isocurvature perturbations, the above non-linear parameters vanish except the first one, yielding

$$f_{NL}^{\zeta,\zeta\zeta} = \frac{5}{6}\left(\frac{3}{2r} + \frac{2}{\xi} - 4 - \frac{r}{\xi^2}\right)\Xi^2, \tag{63}$$

which exactly coincides with the familiar parameter f_{NL}. The amplitude of the non-gaussianities is determined by the three parameters r, ξ and Ξ (note that one recovers the usual prediction of the pure curvaton scenario for $\xi = 1$ and $\Xi = 1$), which take values between 0 and 1. The present constraints on f_{NL}, calculated from WMAP data by assuming purely adiabatic perturbations, are [8]:

$$-10 \leq f_{NL}^{(\text{local})} \leq 74. \tag{64}$$

A sufficiently small r, or even ξ, leads to a significant non-Gaussianity from the adiabatic component, whereas a small Ξ tends to suppress it.

If isocurvature modes are present, however, the five other terms in the reduced bispectrum (57) will also contribute in general. Interestingly, the six functions on the right hand side of (57) have distinct dependences on the l_i, because they involve different combinations of the adiabatic and isocurvature transfer functions. Therefore, this allows in principle to measure, or constrain, independently the corresponding six non-linear parameters from the CMB data. The precise determination of constraints on the $f_{NL}^{I,JK}$ is beyond the scope of the present work, but since all the functions multiplying the $b_{NL}^{I,JK}$ in (57) are of similar amplitude, one can a priori expect the constraints on the $f_{NL}^{I,JK}$ to be of the same order of magnitude as those on f_{NL}^2.

---

[2]Observational constraints on isocurvature non-Gaussianities are given in [19] , for an isocurvature perturbation of the form $S = S_L + f_{NL}^{(iso)} S_L^2$, where S_L is Gaussian. Their non-linear parameter $f_{NL}^{(iso)}$ is related to ours according to $\tilde{f}_{NL}^{S,SS} = 2f_{NL}^{(iso)}\alpha^2$, $\tilde{f}_{NL}^{S,\zeta S} = 2f_{NL}^{(iso)}\alpha^{3/2}|\mathcal{C}|$ and $\tilde{f}_{NL}^{S,\zeta\zeta} = 2f_{NL}^{(iso)}\alpha\mathcal{C}^2$, where $\mathcal{C}$ is the correlation defined in (53).

14

Let us now explore the amplitude of the non-linear parameters in our model. First of all, let us stress that finding significant non-Gaussianities (typically $f_{NL} \sim 10 - 100$) requires, in all cases, a small denominator z_1, i.e. $r \ll 1$, which will thus be assumed below. Second, it is worth noting that all the coefficients are related via the two rules

$$f_{NL}^{I,JS} = \frac{s_1}{z_1} f_{NL}^{I,J\zeta} \ (R1), \qquad f_{NL}^{S,IJ} = \frac{s_2}{z_2} f_{NL}^{\zeta,IJ}, \ (R2) \,. \qquad (65)$$

Therefore, the hierarchy between the parameters can be deduced from the value of the first order ratio

$$\frac{s_1}{z_1} = 3 \left(\frac{f_c}{r} - 1 \right) = \varepsilon_f \sqrt{\frac{\alpha}{\Xi}}, \qquad \varepsilon_f \equiv \mathrm{sgn}(f_c - r), \qquad (66)$$

where we have used (54), as well as the second order ratio s_2/z_2, which is a more complicated expression in general.

We now consider successively the two limits for which the isocurvature bound is satisfied.

4.4.1   Limit $|f_c - r| \ll r$, with $\Xi \simeq 1$ (pure curvaton scenario)

In this case, the isocurvature-adiabatic ratio α must satisfy the observational constraint $\alpha \simeq a_1 \leq 0.0037$, since we are in the fully correlated case. The relevant ratios are given here by

$$\frac{s_1}{z_1} \simeq \varepsilon_f \sqrt{\alpha}, \qquad \frac{s_2}{z_2} \simeq \frac{\varepsilon_f \sqrt{\alpha} - 2\tilde{r}(2 - \tilde{r})}{1 + 2\,\tilde{r}\,(2 - \tilde{r})\,/3} \qquad (67)$$

where we have taken the limit $r = \xi\,\tilde{r} \ll 1$ (although r cannot be smaller than 10^{-2}, to be compatible with observational constraint on f_{NL}). If r is small because $\tilde{r} \ll 1$, then the denominator in the expression for s_2/z_2 reduces to 1. However, if $\xi \ll 1$ while $\tilde{r}$ is of order 1, the full expression for s_2/z_2 is needed.

The value of the first ratio implies that, with respect to $f_{NL}^{\zeta,\zeta\zeta}$, the coefficients $f_{NL}^{\zeta,\zeta S}$ and $f_{NL}^{\zeta,SS}$ are suppressed with factors $\sqrt{\alpha}$ and α, respectively, according to (R1). Analogously the coefficients $f_{NL}^{S,\zeta S}$ and $f_{NL}^{S,SS}$ are suppressed, respectively with factors $\sqrt{\alpha}$ and α, with respect to $f_{NL}^{S,\zeta\zeta}$. By contrast, using (R2), one sees that $f_{NL}^{S,\zeta\zeta}$ could be of the same order of magnitude as $f_{NL}^{\zeta,\zeta\zeta}$, if $\tilde{r} \sim 1$, or suppressed if $\tilde{r}$ is small.

To conclude, in the pure curvaton scenario, it is possible to satisfy the isocurvature constraint and to get measurable non-gaussianities only by assuming a fine-tuning between f_c and r at the percent level. In this situation, only the purely adiabatic parameter is significant, while the other parameters are suppressed, with increasing powers of α.

15

4.4.2 Limit $\Xi \ll 1$

In this limit α must satisfy the constraint $\alpha \simeq a_0 < 0.064$ (uncorrelated case).

In the regime $f_c \ll r \ll 1$, one finds that both ratios s_1/z_1 and s_2/z_2 reduce to (-3), independently of the value of $\tilde{r}$. Therefore, the relation between the non-linear parameters is simply

$$\tilde{f}_{NL}^{\zeta,\zeta\zeta} \simeq \frac{\alpha^2}{54r}, \quad \tilde{f}_{NL}^{I,JK} \simeq (-3)^{I_S} \tilde{f}_{NL}^{\zeta,\zeta\zeta} \qquad (f_c \ll r \ll 1) \tag{68}$$

where I_S is the number of S among the indices (I, JK). This is the result obtained in [17] for $f_c = 0$. For α close to its present upper bound, one sees that detectable non-Gaussianity can be generated with $r \sim 10^{-5}$.

By contrast, in the regime $f_c \gg r$, the purely adiabatic coefficient is strongly suppressed since

$$\tilde{f}_{NL}^{\zeta,\zeta\zeta} \simeq \frac{\alpha^2 r^3}{54 f_c^4} \,. \tag{69}$$

However, the other coefficients are now enhanced with respect to the purely adiabatic coefficient, via the large factors

$$\frac{s_1}{z_1} \simeq 3\frac{f_c}{r}, \qquad \frac{s_2}{z_2} \simeq 3\frac{f_c}{r}(1 - 2f_c)\,. \tag{70}$$

where, for simplicity, we have assumed $\tilde{r} \ll 1$ (the other possibility $\xi \ll 1$ yields a more complicated expression for the second ratio, with a dependence on $\tilde{r}$). The dominant term is therefore the purely isocurvature term

$$\tilde{f}_{NL}^{S,SS} \simeq \alpha^2 \frac{1 - 2f_c}{2f_c} \tag{71}$$

If $f_c \sim 1$, this purely isocurvature non-Gaussianity, although enhanced with respect to all the other contributions, remains negligible since it is suppressed by the very small factor α^2. This was the conclusion reached in [17] (for $f_c = 1$).

However, we now see that this suppression can be compensated if f_c is smaller than α^2. The purely isocurvature parameter and the other ones are then given by

$$\tilde{f}_{NL}^{S,SS} \simeq \frac{\alpha^2}{2f_c}, \quad \tilde{f}_{NL}^{I,JK} \simeq \left(\frac{r}{3f_c}\right)^{I_\zeta} \tilde{f}_{NL}^{S,SS} \qquad (r \ll f_c \ll 1) \tag{72}$$

where I_ζ is the number of ζ among the three indices. One can notice that the amplitude of the purely isocurvature non-Gaussianity does not depend on the

parameter r, but only on α and f_c. For instance, with $\alpha = 0.05$ which satisfies the current observational bound, a value $f_c = 10^{-5}$ yields $\tilde{f}_{NL}^{S,SS} \sim 100$. In such a scenario, one gets observable non-Gaussianity that comes essentially from isocurvature modes, even if the latter are subdominant in the power spectrum.

5   Scenario with two curvatons

We now apply our formalism to the models where two curvatons are present in the early Universe (see e.g. [25, 26, 27]). The curvaton σ will be assumed to decay first, followed later by the curvaton denoted χ.

5.1   First order

At linear order, the decay of the first curvaton can be characterized by the transfer matrix

$$T_{[1]} = \begin{pmatrix} 1 - x_{r1} & x_{c1} & x_{\chi 1} & x_{r1} - x_{c1} - x_{\chi 1} \\ 0 & 1 - f_{c1} & 0 & f_{c1} \\ 0 & 0 & 1 - f_{\chi 1} & f_{\chi 1} \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{73}$$

where the order of the species is (r, c, χ, σ), while the decay of the second curvaton is characterized by the transfer matrix

$$T_{[2]} = \begin{pmatrix} 1 - x_{r2} & x_{c2} & x_{r2} - x_{c2} & 0 \\ 0 & 1 - f_{c2} & f_{c2} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{74}$$

In the above matrices, the definitions of the parameters are analogous to the definitions introduced in (32), i.e. $x_{r1} \equiv f_{r1}/\tilde{\Omega}_1$, $x_{c1} \equiv \Omega_{c1} x_{r1}/4$, $x_{\chi 1} \equiv \Omega_{\chi 1} x_{r1}/4$, etc, and the indices 1 and 2 refer respectively to the first and second decays. We have also allowed the possibility that the first curvaton σ decays into the second curvaton χ, hence the presence of the parameter $f_{\chi 1}$.

The expression of the perturbations for radiation and CDM, after the two transitions, are expressed in terms of the initial perturbations ζ_{B0} via the product of the two transfer matrices given above, i.e.

$$\zeta_A = \sum_B \left(T_{[2]} \cdot T_{[1]} \right)_A^B \zeta_{B0}. \tag{75}$$

At first order, the radiation curvature perturbation, after the second curvaton decay, reads

$$\zeta_{\rm r} = \zeta_{\rm r0} + z_\sigma \, S_{\sigma0} + z_\chi \, S_{\chi0} + z_c \, S_{c0}, \qquad (76)$$

with

$$
\begin{align}
3z_\sigma &= (1 - x_{r2})(x_{r1} - x_{c1} - x_{\chi1}) + f_{c1}x_{c2} + f_{\chi1}(x_{r2} - x_{c2}), \qquad (77) \\
3z_\chi &= (1 - f_{\chi1})(x_{r2} - x_{c2}) + (1 - x_{r2})x_{\chi1}, \qquad (78) \\
3z_c &= (1 - f_{c1})x_{c2} + (1 - x_{r2})x_{c1} \,. \qquad (79)
\end{align}
$$

Combining this expression with that of the CDM curvature perturbation, according to (11), we find that the CDM entropy perturbation is given by

$$S_c = s_\sigma \, S_{\sigma0} + s_\chi \, S_{\chi0} + s_c \, S_{c0}, \qquad (80)$$

with

$$
\begin{align}
s_\sigma &= -3z_\sigma + f_{c1}(1 - f_{c2}) + f_{c2}f_{\chi1}, \qquad (81) \\
s_\chi &= -3z_\chi + f_{c2}(1 - f_{\chi1}), \qquad (82) \\
s_c &= -3z_c + (1 - f_{c1})(1 - f_{c2}). \qquad (83)
\end{align}
$$

For simplicity, we will restrict ourselves, from now on, to the case where $S_{c0} = 0$.

Defining Λ as the ratio between the two curvaton power spectra, such that

$$P_{S_{\chi0}} \equiv \Lambda P_{S_{\sigma0}}, \qquad (84)$$

one easily finds that the ratio between the isocurvature and the adiabatic spectra is given by

$$\alpha = \frac{P_{S_c}}{P_{\zeta_{\rm r}}} = \frac{s_\sigma^2 + \Lambda s_\chi^2}{z_\sigma^2 + \Lambda z_\chi^2} \, \Xi \,, \qquad \Xi \equiv \frac{\lambda_\chi + \lambda_\sigma}{1 + \lambda_\chi + \lambda_\sigma} \qquad (85)$$

where λ_χ and λ_σ are defined as in (51), i.e.

$$\mathcal{P}_{\zeta_{\rm r}} = \mathcal{P}_{\zeta_{\rm r0}} + z_\sigma^2 \mathcal{P}_{S_{\sigma0}} + z_\chi^2 \mathcal{P}_{S_{\chi0}} \equiv (1 + \lambda_\sigma + \lambda_\chi)\mathcal{P}_{\zeta_{\rm r0}}. \qquad (86)$$

The correlation between $\zeta_{\rm r}$ and S_c can be expressed as

$$\mathcal{C} = \frac{z_\sigma s_\sigma + \Lambda z_\chi s_\chi}{\sqrt{(s_\sigma^2 + \Lambda s_\chi^2)(z_\sigma^2 + \Lambda z_\chi^2)}} \, \sqrt{\Xi} \,. \qquad (87)$$

The observational constraints on α impose that at least one of the following conditions must be satisfied:

$$\Xi \ll 1 \quad \text{or} \quad s_\sigma^2 + \Lambda s_\chi^2 \ll z_\sigma^2 + \Lambda z_\chi^2 \,. \tag{88}$$

The first possibility, $\Xi \ll 1$, corresponds to a power spectrum dominated by the inflaton, whereas the second possibility requires special cancellations in (81-82) so that s_σ and s_χ are suppressed.

5.2   Second order

We now consider the perturbations up to the second order, in order to compute the non-Gaussianities. First, let us decompose the curvaton entropy perturbations as in (46), so that

$$S_{\sigma 0} = \hat{S}_\sigma - \frac{1}{4}\hat{S}_\sigma^2 \qquad S_{\chi 0} = \hat{S}_\chi - \frac{1}{4}\hat{S}_\chi^2, \tag{89}$$

where $\hat{S}_\sigma$ and $\hat{S}_\chi$ are two independent Gaussian quantities.

The radiation curvature perturbation and the dark matter entropy perturbation after the second decay, up to second order, are given in our notation by

$$\zeta_{\mathrm{r}} = \zeta_{\mathrm{r}0} + z_\sigma \hat{S}_\sigma + z_\chi \hat{S}_\chi + z_{\sigma\chi}\hat{S}_\sigma \hat{S}_\chi + \frac{1}{2}z_{\sigma\sigma}\hat{S}_\sigma^2 + \frac{1}{2}z_{\chi\chi}\hat{S}_\chi^2 \tag{90}$$

$$S_c = s_\sigma \hat{S}_\sigma + s_\chi \hat{S}_\chi + s_{\sigma\chi}\hat{S}_\sigma \hat{S}_\chi + \frac{1}{2}s_{\sigma\sigma}\hat{S}_\sigma^2 + \frac{1}{2}s_{\chi\chi}\hat{S}_\chi^2 \tag{91}$$

where the coefficients z_σ, z_χ, s_σ and s_χ have already been defined in (77-78) and (81-82), respectively. We do not give explicitly the full expressions for the second order coefficients because they are very lengthy, but they are straightforward to compute by using the general expressions (27-28) with the transfer matrices (73-74).

Let us calculate the reduced bispectrum by using the general expression given in the Appendix. In our model, ignoring the inflaton which does not produce significant non-Gaussianities, the relevant power spectra are independent so that

$$P^{ab}(k) = \begin{pmatrix} 1 & 0 \\ 0 & \Lambda \end{pmatrix} P_{\hat{S}_\sigma} \,, \tag{92}$$

where we furthermore assume that Λ is strictly independent of k (this is indeed the case if the masses of both curvatons are negligible with respect to H during inflation).

As a consequence, the reduced bispectrum can be reduced to the same expression as that already given in Eq. (57) with

$$\beta_l^I(r) \equiv \frac{2}{\pi} \int k^2 dk j_l(kr) g_l^I(k) P_{\hat{S}_\sigma}(k) \tag{93}$$

and the six parameters

$$b_{NL}^{I,JK} \equiv N_{\sigma\sigma}^I N_\sigma^J N_\sigma^K + \Lambda N_{\sigma\chi}^I \left(N_\sigma^J N_\chi^K + N_\chi^J N_\sigma^K\right) + \Lambda^2 N_{\chi\chi}^I N_\chi^J N_\chi^K, \tag{94}$$

where the coefficients N_{ab}^I, which are defined as in (105), can be read off directly from (90) and (91). In complete analogy with the model with one curvaton, to be compared with the standard f_{NL}, these coefficients must be divided by the square of the ratio $P_\zeta/P_{S_{\sigma 0}} = (z_\sigma^2 + \Lambda z_\chi^2)/\Xi$, hence:

$$\tilde{f}_{NL}^{I,JK} \equiv \left(\frac{\Xi}{z_\sigma^2 + \Lambda z_\chi^2}\right)^2 b_{NL}^{I,JK}. \tag{95}$$

The six non-linearity coefficients are thus given by

$$\tilde{f}_{NL}^{\zeta,\zeta\zeta} = \left(\frac{\Xi}{z_\sigma^2 + \Lambda z_\chi^2}\right)^2 \left[z_{\sigma\sigma} z_\sigma^2 + 2\Lambda z_{\sigma\chi} z_\sigma z_\chi + \Lambda^2 z_{\chi\chi} z_\chi^2\right],$$

$$\tilde{f}_{NL}^{\zeta,\zeta S} = \left(\frac{\Xi}{z_\sigma^2 + \Lambda z_\chi^2}\right)^2 \left[z_{\sigma\sigma} z_\sigma s_\sigma + \Lambda z_{\sigma\chi}(z_\sigma s_\chi + z_\chi s_\sigma) + \Lambda^2 z_{\chi\chi} z_\chi s_\chi\right],$$

$$\tilde{f}_{NL}^{\zeta,SS} = \left(\frac{\Xi}{z_\sigma^2 + \Lambda z_\chi^2}\right)^2 \left[z_{\sigma\sigma} s_\sigma^2 + 2\Lambda z_{\sigma\chi} s_\sigma s_\chi + \Lambda^2 z_{\chi\chi} s_\chi^2\right],$$

$$\tilde{f}_{NL}^{S,\zeta\zeta} = \left(\frac{\Xi}{z_\sigma^2 + \Lambda z_\chi^2}\right)^2 \left[s_{\sigma\sigma} z_\sigma^2 + 2\Lambda s_{\sigma\chi} z_\sigma z_\chi + \Lambda^2 s_{\chi\chi} z_\chi^2\right],$$

$$\tilde{f}_{NL}^{S,S\zeta} = \left(\frac{\Xi}{z_\sigma^2 + \Lambda z_\chi^2}\right)^2 \left[s_{\sigma\sigma} z_\sigma s_\sigma + \Lambda s_{\sigma\chi}(s_\sigma z_\chi + s_\chi z_\sigma) + \Lambda^2 s_{\chi\chi} s_\chi z_\chi\right],$$

$$\tilde{f}_{NL}^{S,SS} = \left(\frac{\Xi}{z_\sigma^2 + \Lambda z_\chi^2}\right)^2 \left[s_{\sigma\sigma} s_\sigma^2 + 2\Lambda s_{\sigma\chi} s_\sigma s_\chi + \Lambda^2 s_{\chi\chi} s_\chi^2\right]. \tag{96}$$

In the following we analyze explicitly some limiting cases.

5.3 Various limits

We now explore the parameter space, in order to see whether it is possible to obtain significant non-Gaussianities.

Let us first mention that we have checked that our results agree with those of [27] in the limit where the curvatons decay only into radiation (i.e. $f_{c1} = f_{c2} = f_{\chi 1} = 0$), the dark matter abundance is neglected (i.e. $x_{c1} = x_{c2} = 0$) and the inflaton contribution is ignored (i.e. $\Xi = 1$).

5.3.1 Limit $\Lambda \ll 1$

In this limit where the contributions from the second curvaton are negligible, one finds

$$\alpha \simeq \Xi\,\frac{s_\sigma^2}{z_\sigma^2}, \qquad \tilde{f}_{NL}^{\zeta\zeta\zeta} \simeq \Xi^2 \frac{z_{\sigma\sigma}}{z_\sigma^2}, \tag{97}$$

while the other five non-linear coefficients can be deduced from $\tilde{f}_{NL}^{\zeta\zeta\zeta}$ according to the relations

$$f_{NL}^{I,JS} \simeq \frac{s_\sigma}{z_\sigma} f_{NL}^{I,J\zeta} \qquad f_{NL}^{S,IJ} \simeq \frac{s_{\sigma\sigma}}{z_{\sigma\sigma}} f_{NL}^{\zeta,IJ} . \tag{98}$$

The quantity α is constrained by observations to be small, which requires either $\Xi \ll 1$ or $|s_\sigma| \ll |z_\sigma|$.

First possibility: $\Xi \ll 1,$ while $|z_\sigma| \sim |s_\sigma|$.
    This leads to a suppression of all the non-Gaussianity coefficients by a factor Ξ^2. However, the coefficients $\tilde{f}_{NL}^{\zeta,JK}$ can still be significant if the ratio $z_{\sigma\sigma}/z_\sigma^2$ can compensate the Ξ^2 suppression (similarly for the $f_{NL}^{S,JK}$ if $s_{\sigma\sigma}/z_\sigma^2$ compensates the Ξ^2 suppression).
    Let us consider a specific example, with the simplifying assumptions

$$x_{c1} = x_{c2} = f_{\chi 1} = x_{\chi 1} = 0 , \tag{99}$$

that is, we neglect the energy fraction of dark matter and assume that the curvaton σ does not decay into χ and that χ is subdominant when σ decays. Under these assumptions, $z_\sigma = x_{r1}(1-x_{r2})/3$ and we further assume $f_{c1} \ll z_\sigma$ so that $s_\sigma \simeq -3z_\sigma$. In the two limits $x_{r1} = \tilde{r}_1 \xi_1 \ll 1$ and $(1 - x_{r2}) \ll 1$, z_σ is small and the adiabatic non-Gaussianity behaves as

$$\tilde{f}_{NL}^{\zeta,\zeta\zeta} = \frac{1}{1 - x_{r2}} \left[\tilde{f}_{NL1}^{\zeta,\zeta\zeta} + \frac{x_{r2}}{1 - x_{r2}} \left(\frac{3}{2} + x_{r2}\, \tilde{f}_{NL2}^{\zeta,\zeta\zeta} \right) \right] , \tag{100}$$

where $\tilde{f}_{NL\,1,2}^{\zeta,\zeta\zeta}$ correspond to single-curvaton coefficient, equation (63), but calculated with the parameters $\xi_{1,2}$ and $x_{r1,r2}$ respectively.
    If we assume $x_{r2} \ll 1$, the above equation corresponds to the single-curvaton result (68). The other coefficients also follow the relations given in (68), since $s_{\sigma\sigma}/z_{\sigma\sigma} = -3$ with the assumptions (99) and $f_{c1} \ll 1$, and are thus of comparable magnitude.

Second possibility: $|s_\sigma| \ll |z_\sigma|$

When a small α is the consequence of $|s_\sigma| \ll |z_\sigma|$, one sees from the first relation in (98) that all the $f_{NL}^{I,JS}$ are strongly suppressed with respect to $f_{NL}^{I,J\zeta}$. However, the two coefficients $f_{NL}^{I,\zeta\zeta}$ can still be important if $|z_\sigma|$ is sufficiently small. By examining (77) and (81), one sees that getting $|s_\sigma| \ll |z_\sigma| \ll 1$ requires some fine-tuning between the coefficients, which we now discuss.

In order to get $|z_\sigma| \ll 1$, the first possibility is that the first curvaton is subdominant, i.e. $x_{r1} = \mathcal{O}(\epsilon)$, where ϵ is some small number (we neglect x_{c2} which must be small because we are deep in the radiation era), which then requires either $x_{r2} = \mathcal{O}(\epsilon)$ or $f_{\chi 1} = \mathcal{O}(\epsilon)$. The second possibility is that the second curvaton dominates at decay, i.e. $x_{r2} = 1 - \mathcal{O}(\epsilon)$, which also requires that $f_{\chi 1} = \mathcal{O}(\epsilon)$. Then, to obtain $|s_\sigma| \ll |z_\sigma|$, the terms of the right hand side of (81), which are of order ϵ must compensate each other so that their sum is at most of order $\mathcal{O}(\alpha \epsilon)$, which necessitates some special relation between the f_A and the x_A.

If we consider again the assumptions (99) and neglect f_{c2}, one finds that the fine-tuning condition to get $|s_\sigma| \ll |z_\sigma|$ is

$$f_{c1} - x_{r1}(1 - x_{r2}) \leq \mathcal{O}(\alpha\epsilon). \tag{101}$$

The adiabatic parameter $\tilde{f}_{NL}^{\zeta,\zeta\zeta}$ is given in equation (100), with now $\Xi \sim 1$, and its value is of order 10 when $\epsilon \sim x_{r1}(1 - x_{r2}) \sim 0.1$. Since $s_{\sigma\sigma}/z_{\sigma\sigma} \simeq -3 + \mathcal{O}(f_{c1}/x_{r1}(1 - x_{r2}))$, we also have $\tilde{f}_{NL}^{S,\zeta\zeta} \sim \tilde{f}_{NL}^{\zeta,\zeta\zeta}$.

Note that a significant non-Gaussianity generated by a dominant curvaton ($x_{r2} = 1 - \mathcal{O}(\epsilon)$) has already been pointed out in [27], but we see here that satisfying the isocurvature bound requires additional constraints on the branching ratios of the curvatons.

5.3.2 Limit $\Lambda \gg 1$

In this limit, one obtains

$$\alpha \sim \Xi\, \frac{s_\chi^2}{z_\chi^2}, \qquad \tilde{f}_{NL}^{\zeta\zeta\zeta} \sim \Xi^2\, \frac{z_{\chi\chi}}{z_\chi^2}, \qquad f_{NL}^{I,JS} \simeq \frac{s_\chi}{z_\chi} f_{NL}^{I,J\zeta}, \qquad f_{NL}^{S,IJ} \simeq \frac{s_{\chi\chi}}{z_{\chi\chi}} f_{NL}^{\zeta,IJ}. \tag{102}$$

By comparing with (97) and (98), one sees that the analysis is analogous to the previous case, by replacing z_σ, $z_{\sigma\sigma}$, s_σ and $s_{\sigma\sigma}$ by z_χ, $z_{\chi\chi}$, s_χ and $s_{\chi\chi}$, respectively.

When the curvaton contribution to the power spectrum is not negligible, significant non-Gaussianity, while satisfying the isocurvature bound, is obtained when $|s_\chi| \ll |z_\chi| \ll 1$. This constraint is satisfied if one assumes $f_{\chi 1} = 1 - \mathcal{O}(\epsilon)$, which means that the second curvaton is created mainly by

the decay of the first, while $x_{r2} = 1 - \mathcal{O}(\epsilon)$, $x_{\chi 1} \lesssim \mathcal{O}(\epsilon)$ and $f_{c2} = 1 - \mathcal{O}(\epsilon)$. Other possibilities exist but require some fine-tuning between the parameters, in analogy with the previous analysis in the case $\Lambda \ll 1$.

5.3.3 Intermediate values of Λ

In this case, one must satisfy simultaneously the constraints $|s_\sigma| \ll |z_\sigma|$ and $|s_\chi| \ll |z_{\tilde{\chi}}|$, due to the isocurvature bound. The relative strength of the different f_{NL} coefficients cannot be expressed in such a simple form as in (98), but it will be determined again by the ratios s_σ/z_σ, s_χ/z_χ, $s_{\sigma\sigma}/z_{\sigma\sigma}$ and $s_{\chi\chi}/z_{\chi\chi}$.

In order to get also a significant non-Gaussianity, we look for parameter values such that

$$z_\sigma, z_\chi \sim \mathcal{O}(\epsilon), \qquad s_\sigma, s_\chi \lesssim \mathcal{O}(\alpha\,\epsilon). \tag{103}$$

These constraints can be satisfied by fine-tuning the parameters. Solving $s_\sigma \simeq 0$ and $s_\chi \simeq 0$ for the two parameters f_{c1} and f_{c2} yields

$$f_{c1} \simeq \frac{(x_{r1} - x_{c1})(1 - f_{\chi 1}) - x_{\chi 1}}{1 - f_{\chi 1} - x_{\chi 1}}, \qquad f_{c2} \simeq x_{r2} - x_{c2} + \frac{1 - x_{r2}}{1 - f_{\chi 1}}\, x_{\chi 1}. \tag{104}$$

The observational constraint on the isocurvature power spectrum is satisfied if these two fine-tuning relations hold simultaneously, at the level $\mathcal{O}(\alpha\,\varepsilon)$. Using these relations, one finds interesting non-Gaussianity for the following set of parameters: $x_{r1} = \mathcal{O}(\epsilon)$, $x_{r2} = \mathcal{O}(\epsilon)$, $x_{\chi 1} = \mathcal{O}(\alpha\,\epsilon)$, $f_{c1} = x_{r1} - x_{c1} + \mathcal{O}(\alpha\,\epsilon)$, $f_{c2} = x_{r2} + \mathcal{O}(\alpha\,\epsilon)$, with negligible values for x_{c2}. In this scenario, both curvatons are subdominant at their decay and the fraction of produced dark matter is fine-tuned.

6 Conclusions

In this work, we have introduced a systematic treatment in order to compute the evolution of linear and non-linear cosmological perturbations in a cosmological transition due to the decay of some particle species. Our main results can be summarized as follows.

At the linear level, the evolution of all individual curvature perturbations can be expressed in terms of a transfer matrix, whose coefficients depend on background quantities, such as the relative abundances of the fluids at the decay, their equation of state parameters and the relative decay branching ratios [see Eqs (21-25)]. At the non-linear level, the post-decay curvature perturbations can also be given in terms of the pre-decay perturbations quite generally, and we have presented explicitly these relations at second order [see

Eqs (27-28)]. We have then applied our general formalism to two specific examples.

The first example is the mixed curvaton-inflaton scenario in which we allow the dark matter to be created both before *and* during the curvaton decay. We find, in particular, the remarkable result that it is possible to obtain *isocurvature dominated* non-Gaussianities with, as required by the CMB measurements, an adiabatic dominated power spectrum.

In the second example, we have studied scenarios with several curvaton-like fields and obtained results that generalize previous works on two-curvaton scenarios by taking into account the various decay products of the curvatons. We have explored the parameter space to see whether it is possible to find significant non-Gaussianity while satisfying the isocurvature bound in the power spectrum. We have found that several such regions exist, but often at the price of a fine-tuning between the parameters.

In the presence of isocurvature modes, which can be correlated with the adiabatic modes, non-Gaussianity of the local type is much richer than in the purely adiabatic case and we have shown that the angular bispectrum is the sum of six different contributions. As a consequence, in addition to the traditional f_{NL} (adiabatic) parameter, we have identified five new non-linear parameters that must be taken into account: one purely isocurvature parameter and four correlated parameters. We have computed these parameters in the two models we have investigated.

Beyond the two examples considered in this work, our formalism can be used as a general toolbox to study systematically the cosmological constraints, arising from linear perturbations and from non-Gaussianities, for particle physics models and their associated cosmological scenarios.

A   Angular bispectrum

We consider several observable quantities X^I, like ζ and S_c, which depend on "primordial" scalar fields ϕ^a, whose perturbations are generated during inflation. Up to second order, one can formally write

$$X^I = N_a^I \phi^a + \frac{1}{2} N_{ab}^I \phi^a \phi^b + \dots \qquad (105)$$

We assume that the ϕ^a are Gaussian random fields, with the two-point correlation functions

$$\langle \phi^a(\vec{k})\phi^b(\vec{k}')\rangle = (2\pi)^3\, P^{ab}(k)\,\delta(\vec{k}+\vec{k}')\,. \tag{106}$$

We then define the bispectra of the X^I by

$$\langle X^I_{\vec{k}_1} X^J_{\vec{k}_2} X^K_{\vec{k}_3}\rangle = (2\pi)^3\delta(\Sigma_i\vec{k}_i)B^{IJK}(k_1,k_2,k_3)\,. \tag{107}$$

Substituting the decomposition (105) into the left hand side, and using (106), one finds

$$
\begin{aligned}
B^{IJK}(k_1,k_2,k_3) = & N^I_a N^J_b N^K_{cd} P^{ac}(k_1)P^{bd}(k_2) + N^I_a N^J_{bc} N^K_d P^{ab}(k_1)P^{cd}(k_3)\\
&+ N^I_{ab} N^J_c N^K_d P^{ac}(k_2)P^{bd}(k_3).
\end{aligned} \tag{108}
$$

As shown in [37], the angular bispectrum can be expressed in terms of the "reduced bispectrum" $b_{l_1l_2l_3}$, according to the expression

$$\langle a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}\rangle = \mathcal{G}^{m_1m_2m_3}_{l_1l_2l_3} b_{l_1l_2l_3}, \tag{109}$$

where

$$\mathcal{G}^{m_1m_2m_3}_{l_1l_2l_3} \equiv \int d^2\hat{\mathbf{n}}\, Y_{l_1m_1}(\hat{\mathbf{n}})\, Y_{l_2m_2}(\hat{\mathbf{n}})\, Y_{l_3m_3}(\hat{\mathbf{n}}) \tag{110}$$

is the Gaunt integral.

The next step is to express the reduced bispectrum in terms of the generalized bispectra B^{IJK}, using the fact that the observable quantity is related to the initial perturbations X^I via a transfer function $g^I_l(k)$, so that

$$a_{lm} = 4\pi(-i)^l \int \frac{d^3\vec{k}}{(2\pi)^3}\left(\sum_I X^I(\vec{k})g^I_l(k)\right) Y^*_{lm}(\hat{\vec{k}}). \tag{111}$$

Substituting the above expression in the left hand side of (109), one finally obtains

$$b_{l_1l_2l_3} = 3\sum_{I,J,K} N^I_{ab}N^J_c N^K_d \int_0^\infty r^2 dr\, \tilde{\beta}^I_{(l_1}(r)\beta^{J,ac}_{l_2}(r)\beta^{K,bd}_{l_3)}(r), \tag{112}$$

with

$$\tilde{\beta}^I_l(r) \equiv \frac{2}{\pi}\int k^2 dk\, j_l(kr)g^I_l(k), \qquad \beta^{I,ab}_l(r) \equiv \frac{2}{\pi}\int k^2 dk\, j_l(kr)g^I_l(k)P^{ab}(k)\,. \tag{113}$$

Note that the "reduced" bispectrum is symmetric with respect to permutations of the indices l_1, l_2 and l_3 (we use the standard notation: $(l_1l_2l_3) \equiv [l_1l_2l_3 + 5\,\text{perms}]/3!$).

In the simplest case, one considers only the adiabatic mode, ζ (or the gravitational potential Φ), which is assumed to depend on a single "primordial" Gaussian field. In this case, where both the indices I and a take a single value, one recovers immediately the familiar result of [37]. Our general expression also includes the particular situation considered in [19], where $\zeta = \phi + (3/5)f_{NL}\phi^2$ and $S = \eta + f_{NL}^{\rm iso}\eta^2$, ϕ and η being Gaussian variables.

References

[1] D. Langlois, Lect. Notes Phys. **800**, 1 (2010) [arXiv:1001.5259 [astro-ph.CO]].

[2] A. D. Linde and V. F. Mukhanov, Phys. Rev. D **56** (1997) 535 [arXiv:astro-ph/9610219]. K. Enqvist and M. S. Sloth, Nucl. Phys. B **626**, 395 (2002) [arXiv:hep-ph/0109214]; D. H. Lyth and D. Wands, Phys. Lett. B **524**, 5 (2002) [arXiv:hep-ph/0110002]; T. Moroi and T. Takahashi, Phys. Lett. B **522**, 215 (2001) [Erratum-ibid. B **539**, 303 (2002)] [arXiv:hep-ph/0110096].

[3] D. H. Lyth, C. Ungarelli and D. Wands, Phys. Rev. D **67**, 023503 (2003) [arXiv:astro-ph/0208055].

[4] D. H. Lyth and D. Wands, Phys. Rev. D **68**, 103516 (2003) [arXiv:astro-ph/0306500].

[5] M. Lemoine and J. Martin, Phys. Rev. D **75**, 063504 (2007) [arXiv:astro-ph/0611948].

[6] M. Lemoine, J. Martin and G. Petit, Phys. Rev. D **78**, 063516 (2008) [arXiv:0802.1601 [astro-ph]].

[7] M. Lemoine, J. Martin and J. Yokoyama, Phys. Rev. D **80**, 123514 (2009) [arXiv:0904.0126 [astro-ph.CO]].

[8] E. Komatsu *et al.*, arXiv:1001.4538 [astro-ph.CO].

[9] D. Wands, Class. Quant. Grav. **27**, 124002 (2010) [arXiv:1004.0818 [astro-ph.CO]].

[10] C. T. Byrnes and K. Y. Choi, arXiv:1002.3110 [astro-ph.CO].

[11] M. Zaldarriaga, Phys. Rev. D **69**, 043508 (2004) [arXiv:astro-ph/0306006].

[12] F. Vernizzi, Phys. Rev. D **69**, 083526 (2004) [arXiv:astro-ph/0311167].

[13] D. Langlois and L. Sorbo, JCAP **0908**, 014 (2009) [arXiv:0906.1813 [astro-ph.CO]].

[14] M. Beltran, Phys. Rev. D **78**, 023530 (2008) [arXiv:0804.1097 [astro-ph]].

[15] M. Kawasaki, K. Nakayama, T. Sekiguchi, T. Suyama and F. Takahashi, JCAP **0811**, 019 (2008) [arXiv:0808.0009 [astro-ph]].

[16] M. Kawasaki, K. Nakayama and F. Takahashi, JCAP **0901**, 002 (2009) [arXiv:0809.2242 [hep-ph]].

[17] D. Langlois, F. Vernizzi and D. Wands, JCAP **0812**, 004 (2008) [arXiv:0809.4646 [astro-ph]].

[18] M. Kawasaki, K. Nakayama, T. Sekiguchi, T. Suyama and F. Takahashi, JCAP **0901**, 042 (2009) [arXiv:0810.0208 [astro-ph]].

[19] C. Hikage, K. Koyama, T. Matsubara, T. Takahashi and M. Yamaguchi, Mon. Not. Roy. Astron. Soc. **398**, 2188 (2009) [arXiv:0812.3500 [astro-ph]].

[20] E. Kawakami, M. Kawasaki, K. Nakayama and F. Takahashi, JCAP **0909**, 002 (2009) [arXiv:0905.1552 [astro-ph.CO]].

[21] T. Takahashi, M. Yamaguchi and S. Yokoyama, Phys. Rev. D **80**, 063524 (2009) [arXiv:0907.3052 [astro-ph.CO]].

[22] D. Langlois and F. Vernizzi, Phys. Rev. D **70**, 063522 (2004) [arXiv:astro-ph/0403258].

[23] F. Ferrer, S. Rasanen and J. Valiviita, JCAP **0410**, 010 (2004) [arXiv:astro-ph/0407300].

[24] G. Lazarides, R. R. de Austri and R. Trotta, Phys. Rev. D **70**, 123527 (2004) [arXiv:hep-ph/0409335].

[25] K. Hamaguchi, M. Kawasaki, T. Moroi and F. Takahashi, Phys. Rev. D **69**, 063504 (2004) [arXiv:hep-ph/0308174].

[26] K. Y. Choi and J. O. Gong, JCAP **0706**, 007 (2007) [arXiv:0704.2939 [astro-ph]].

[27] H. Assadullahi, J. Valiviita and D. Wands, Phys. Rev. D **76**, 103003 (2007) [arXiv:0708.0223 [hep-ph]].

[28] Q. G. Huang, JCAP **0809**, 017 (2008) [arXiv:0807.1567 [hep-th]].

[29] D. Langlois and F. Vernizzi, Phys. Rev. Lett. **95**, 091303 (2005) [arXiv:astro-ph/0503416].

[30] D. Langlois and F. Vernizzi, Phys. Rev. D **72**, 103501 (2005) [arXiv:astro-ph/0509078].

[31] D. Langlois and F. Vernizzi, Class. Quant. Grav. **27**, 124007 (2010) [arXiv:1003.3270 [astro-ph.CO]].

[32] D. Langlois and F. Vernizzi, JCAP **0602**, 014 (2006) [arXiv:astro-ph/0601271].

[33] D. H. Lyth, K. A. Malik and M. Sasaki, JCAP **0505**, 004 (2005) [arXiv:astro-ph/0411220].

[34] S. Gupta, K. A. Malik and D. Wands, Phys. Rev. D **69**, 063513 (2004) [arXiv:astro-ph/0311562].

[35] D. Langlois, Phys. Rev. D **59**, 123512 (1999) [arXiv:astro-ph/9906080].

[36] D. Langlois and A. Riazuelo, Phys. Rev. D **62**, 043504 (2000) [arXiv:astro-ph/9912497].

[37] E. Komatsu, D. N. Spergel, Phys. Rev. **D63**, 063002 (2001). [astro-ph/0005036].